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FINAL REPORT

GRANT NO. NSG 1071

"STATISTICAL SUPPORT
FOR THE ATL PROGRAM"

July 1, 1974 - June 15, 1976

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INTRODUCTION AND SUMMARY

The investigations "Statistical Support for the ATL" were done in essentially three phases:

Phase I: Preliminary survey of all experiments listed in Technical Report NASA TMX - 2813;

Phase II: Interviews with Principal Investigators of some experiments;

Phase III: Development of statistical designs and analysis for the experiment "Colony Growth in Zero Gravity".

This report gives detailed statistical experimental designs for various numbers of organisms and agar solutions pertinent to the experiment. 7-day and 30-day missions have been considered. For the designs listed, the statistical analysis of the observations obtained on the space shuttle has been outlined.

PHASE I:

In order to determine which experiments from the ATL program would lend themselves to statistical applications, we considered and studied in as much detail as possible all the experiments as described in the technical report NASA TMX - 2813. We tried to determine from the descriptions given there, whether an experiment could possibly benefit from a statistical design as to how the experiment should be performed on the space shuttle and/or to what extent more detailed statistical analysis might help in the interpretation of the data once they are collected.

PHASE II

After completion of Phase I, it was decided, in cooperation with Dr. Janet W. Campbell, Technical Monitor of the grant, to consider and discuss in more detail the following four experiments:

1. "Lidar Measurements of Cirrus Clouds and Lower Stratospheric Aerosols", Dr. Ellis Remsburg, Principal Investigator.
2. "Colony Growth in Zero Gravity", Dr. Judd Wilkins, Principal Investigator.
3. "Microwave Radiometer Measurements", Dr. Calvin Swift, Principal Investigator.
4. "Environmental Effects on Nonmetallic Materials", Mr. Wayne S. Slemp, Principal Investigator.

The purpose of these discussions was to get a clearer understanding of the experiments and thereby determine to what extent these experiments or parts of them were statistical in nature. We looked for applications of statistical experimental design techniques for the data gathering phase and statistical analysis techniques for the evaluation phase of each experiment.

Based upon the information obtained we found that, at the present time, only the experiment "Colony Growth in Zero Gravity" would call for statistical input. Our findings and contributions concerning this experiment are reported in detail under Phase III.

PHASE III.

The objectives and description of the experiment, "Colony growth in zero gravity" are given in Technical Report NASA TM X-2813, p. 109-111. The only difference between the equipment described there and the one actually used is that there will be 10 culture tubes instead of 8.

The following basic situations were considered:

1. A 7-day mission with a 5-day observation period;
2. a 30 -day mission with a 25-day observation period.

Since the principal investigator wants to study the growth patterns of different organisms in different agar solutions we considered accordingly the following possibilities: For the 7-day mission:

- 1.1 2 organisms, 5 agar solutions
- 1.2 3 organisms, 5 agar solutions
- 1.3 4 organisms, 5 agar solutions
- 1.4 5 organisms, 5 agar solutions;

and for the 30 -day mission:

- 2.1 6 organisms, 5 agar solutions
- 2.2 7 organisms, 5 agar solutions
- 2.3 8 organisms, 5 agar solutions
- 2.4 9 organisms, 5 agar solutions
- 2.5 10 organisms, 5 agar solutions.

A. THE STATISTICAL DESIGNS:

For each combination 1.1 - 1.4 and 2.1 - 2.5 we have constructed appropriate statistical experimental designs. These are listed in Tables 1.1 - 1.4 and 2.1 - 2.5, respectively.

In Tables 1.1 - 1.4 we have listed the actual (randomized) assignments of all organisms-agar solution (O, A) combinations to the positions on the culture tube rack for each of the five days of experimentation. The incomplete block designs were constructed in such a way that they have the following properties: (i) they are nearly balanced in the sense that each (O, A)-combination appears with almost equal frequency, (ii) they are pseudo-globally connected, i.e. give as much information as possible with respect to comparisons of different (O, A)-combinations, and (iii) they are nearly resistant in the sense that the effect of the loss of the data for any one day is minimized.

In Tables 2.1 - 2.5 we give the arrangement of (O, A)-combinations for each day of experimentation in a slightly different way. Because of the larger number of (O, A)-combinations and hence the increase in "complexity" of the experiment, it seems preferable to use any given organism together with the five agar solutions on the same day. As a result, only two organisms are used each day, indicated by "X" in the appropriate table. For the actual experiment the ten designated (O, A)-combinations have to be randomly assigned to the ten positions on the culture tube rack. These incomplete block designs are (i) nearly balanced over the whole duration of the experiment, (ii) nearly balanced over each five-day period, (iii) pseudo-globally connected, and (iv) nearly resistant.

B. THE ANALYSIS

Each of the designs given under A is an incomplete block design for which an appropriate model is of the form

$$y_{ijk} = \mu + \tau_{ij} + d_k + \varepsilon_{ijk}, \quad (1)$$

$$(i = 1, 2, \dots, p; j = 1, 2, \dots, 5; k = 1, 2, \dots, d)$$

where y_{ijk} = response of the i-th organism in the j-th agar solution obtained on the k-th day of experimentation (we suggest that some form of index be developed to characterize the growth patterns numerically, denoted by y),

μ = overall mean

τ_{ij} = effect of the i-th organism and j-th agar solution combination,

d_k = effect of the k-th day

ϵ_{ijk} = random error.

Let r_{ij} denote the number of times that the combination of the i-th organism and the j-th agar solution occur in the experiment and let

$$R = \begin{pmatrix} r_{11} & & & & 0 \\ r_{12} & & & & \\ \ddots & & & & \\ & r_{15} & & & \\ & & \ddots & & \\ 0 & & & & r_{p5} \end{pmatrix},$$

where p denotes the number of organisms used in a particular experiment. Furthermore, let N denote the incidence matrix. N is a $p \times d$ matrix, where d denotes the number of days of experimentation, consisting of 1's and 0's, a 1 in the $[(i,j),k]$ th position indicating that the combination of the i-th organisms and the j-th agar solution is being observed on the k-th day of experimentation, a 0 indicating that the particular $(0,A)$ -combination is not being observed on that day.

The reduced normal equations for estimating the "treatment" effects τ_{ij} are then given as

$$(R - \frac{1}{10} NN') \hat{\tau} = Q , \quad (2)$$

where

$$\hat{\tau}' = (\hat{\tau}_{11}, \hat{\tau}_{12}, \dots, \hat{\tau}_{15}, \dots, \hat{\tau}_{p5})$$

and

$$Q' = (Q_{11}, Q_{12}, \dots, Q_{15}, \dots, Q_{p5})$$

with

$$\begin{aligned} Q_{ij} &= (\text{Sum of all observations for } (i,j)\text{-th treatment combination}) \\ &\quad - \frac{1}{10} (\text{Sum of all day totals on which the } (i,j)\text{-th} \\ &\quad \text{combination is being observed}). \end{aligned}$$

A solution to the equations (2) is given by

$$\hat{\tau} = (R - \frac{1}{10} NN' + J)^{-1} Q , \quad (3)$$

where J is a $5p \times 5p$ matrix of unity elements. Let

$$(R - \frac{1}{10} NN' + J)^{-1} = V \quad (4)$$

and denote a typical element of V by $v_{ij, i'j'}$, where $i, i' = 1, 2, \dots, p$; $j, j' = 1, 2, 3, 4, 5$.

Since in the present study the "treatments" represent combinations of organisms and agar-solutions, it is appropriate to take this factorial structure into account by partitioning τ_{ij} in the following way:

$$\tau_{ij} = \theta_i + a_j + (0a)_{ij} ,$$

where θ_i = effect of i -th organism

a_j = effect of j -th agar solution,

$(\theta a)_{ij}$ = interaction between i -th organism and j -th agar solution.

In terms of the solutions (3) these effects can be estimated as

$$\hat{\theta}_i = \frac{1}{5} \sum_{j=1}^5 \tau_{ij} \quad (5)$$

$$\hat{a}_j = \frac{1}{p} \sum_{i=1}^p \tau_{ij} \quad (6)$$

$$(\hat{\theta a})_{ij} = \hat{\tau}_{ij} - \hat{\theta}_i - \hat{a}_j \quad (7)$$

The following comparisons are of interest:

- (i) Growth pattern of i -th organism versus growth pattern of i -th organism;
- (ii) growth pattern of i -th organism in j -th agar solution versus growth pattern of i -th organism in j' -th agar solution;
- (iii) growth pattern of i -th organism in j -th agar solution versus growth pattern of i' -th organism in j -th agar solution.

Using (5), (6), (7) these comparisons are estimated as

$$(i) \hat{\theta}_i - \hat{\theta}_{i'} \quad (8)$$

$$(ii) \hat{a}_j + (\hat{\theta a})_{ij} - \hat{a}_{j'} - (\hat{\theta a})_{i'j'} = \hat{\tau}_{ij} - \hat{\tau}_{i'j'} \quad (9)$$

$$(iii) \hat{\theta}_i + (\hat{\theta a})_{ij} - \hat{\theta}_{i'} - (\hat{\theta a})_{i'j} = \hat{\tau}_{ij} - \hat{\tau}_{i'j} \quad (10)$$

The variances of these comparisons are given by

$$(i) \text{var}(\hat{\theta}_i - \hat{\theta}_{i'}) = \frac{1}{25} \left[\sum_{j,j'} v_{ij,ij'} + \sum_{j,j'} v_{i'j,i'j'} - 2 \sum_{j,j'} v_{ij,i'j'} \right] \sigma^2 \quad (11)$$

$$(ii) \text{ var}(\hat{\tau}_{ij} - \hat{\tau}_{ij'}) = [v_{ij,ij} + v_{ij',ij'} - 2v_{ij,ij'}] \sigma^2 \quad (12)$$

$$(iii) \text{ var}(\hat{\tau}_{ij} - \hat{\tau}_{i'j}) = [v_{ij,ij} + v_{ij,i'j} - 2v_{ij,i'j}] \sigma^2 \quad (13)$$

In (11), (12), (13) σ^2 has to be estimated from the usual intra-block analysis of variance table for incomplete block designs. For the designs given in Tables 1.2 - 1.4, the elements $v_{ij,i'j}$ are listed in Tables 3.2 - 3.4, respectively, where the rows and columns are labeled in the order (1,1), (1,2), ..., (1,5), (2,1), (2,2), ..., (2,5), etc. The design of Table 1.1 is, of course, a randomized complete block design, and hence all diagonal elements $v_{ij,ij} = 1/5$ and all off-diagonal elements $v_{ij,i'j} = 0$. For the designs given in Tables 2.1 - 2.5 the elements $v_{ij,i'j}$ will have to be computed, using the definition of V as given in (4).

1. EXPERIMENTAL DESIGNS FOR SEVEN-DAY MISSION.

1.1 2 ORGANISMS, 5 AGAR SOLUTIONS

DAY 1: (2,1) (2,3) (1,3) (2,2) (2,5)
(1,1) (1,4) (2,4) (1,2) (1,5)

DAY 2: (1,3) (1,1) (1,2) (2,1) (2,2)
(2,4) (2,3) (2,5) (1,5) (1,4)

DAY 3: (1,2) (2,3) (2,4) (2,1) (2,5)
(1,1) (1,3) (2,2) (1,5) (1,4)

DAY 4: (2,5) (2,3) (1,3) (1,1) (2,2)
(1,5) (2,4) (1,2) (1,4) (2,1)

DAY 5: (1,1) (1,4) (1,2) (2,2) (1,5)
(2,4) (2,1) (2,3) (1,3) (2,5)

1.2 3 ORGANISMS, 5 AGAR SOLUTIONS

DAY 1: (1,3) (2,5) (1,5) (2,1) (1,4)
(2,4) (1,1) (1,2) (2,2) (2,3)

DAY 2: (3,1) (1,3) (3,2) (1,4) (3,5)
(1,1) (1,2) (1,5) (3,3) (3,4)

DAY 3: (2,1) (3,4) (3,2) (2,2) (3,5)
(3,3) (2,3) (2,5) (3,1) (2,4)

DAY 4: (2,1) (3,4) (2,3) (3,2) (3,1)
(1,1) (3,3) (1,3) (1,2) (2,2)

DAY 5: (1,3) (3,4) (2,5) (2,3) (2,4)
(3,3) (3,5) (3,2) (1,4) (1,5)

1.3 4 ORGANISMS, 5 AGAR SOLUTIONS

DAY 1: (3,1) (2,1) (4,3) (4,2) (2,2)
(3,3) (1,2) (4,1) (3,2) (1,1)

DAY 2: (1,5) (1,4) (1,3) (4,4) (2,5)
(3,4) (2,3) (2,4) (3,5) (4,5)

DAY 3: (1,1) (1,3) (2,1) (3,1) (3,3)
(1,2) (4,1) (4,2) (3,2) (2,2)

DAY 4: (4,4) (2,3) (4,3) (1,5) (3,4)
(3,5) (4,5) (2,5) (1,4) (2,4)

DAY 5: (2,2) (1,3) (3,3) (4,4) (4,3)
(1,4) (3,2) (3,4) (2,3) (1,2)

1.4 5 ORGANISMS, 5 AGAR SOLUTIONS

DAY 1: (1,2) (4,1) (1,1) (5,1) (3,1)
(4,2) (2,2) (3,2) (5,2) (2,1)

DAY 2: (3,3) (4,2) (1,3) (4,3) (5,3)
(5,2) (2,2) (1,2) (2,3) (3,2)

DAY 3: (5,4) (2,3) (3,4) (4,3) (1,4)
(5,3) (3,3) (2,4) (4,4) (1,3)

DAY 4: (3,4) (2,5) (5,4) (1,4) (2,4)
(4,5) (4,4) (3,5) (5,5) (1,5)

DAY 5: (5,1) (1,1) (4,1) (2,5) (3,5)
(3,1) (4,5) (2,1) (1,5) (5,5)

2. EXPERIMENTAL DESIGNS FOR 30-DAY MISSION

2.1 6 Organisms, 5 Agar Solutions

O-A Combi- nations	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
DAY																									
(1,1)-(1,5)	X			X	X	X				X			X		X		X								
(2,1)-(2,5)	X			X	X	X				X	X		X		X		X								
(3,1)-(3,5)	X			X	X	X				X	X		X		X		X								
(4,1)-(4,5)	X			X	X	X				X	X		X		X		X								
(5,1)-(5,5)	X			X	X	X				X	X		X		X		X								
(6,1)-(6,5)	X			X	X	X				X	X		X		X		X								

2.2 7 ORGANISMS, 5 AGAR SOLUTIONS

2.3 8 ORGANISMS, 5 AGAR SOLUTIONS

0-A Combi- nations	DAY																							
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
(1,1)-(1,5)	x					x		x			x					x			x					x
(2,1)-(2,5)	x					x		x			x					x			x					x
(3,1)-(3,5)	x					x		x			x					x			x					x
(4,1)-(4,5)		x				x		x			x					x			x					x
(5,1)-(5,5)	x					x		x			x					x		x	x					x
(6,1)-(6,5)	x					x		x			x					x		x	x					x
(7,1)-(7,5)		x				x		x			x					x		x	x					x
(8,1)-(8,5)		x				x		x			x					x		x	x					x

2.4 9 ORGANISMS, 5 AGAR SOLUTIONS

O-A
Combina-
tion DAY

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
(1,1)-(1,5)	X				X	X				X					X										
(2,1)-(2,5)	X					X	X								X					X					
(3,1)-(3,5)	X						X								X		X			X				X	
(4,1)-(4,5)	X							X							X			X		X					
(5,1)-(5,5)	X								X						X			X		X				X	
(6,1)-(6,5)	X								X						X			X		X					
(7,1)-(7,5)	X								X						X			X		X					
(8,1)-(8,5)	X								X						X			X		X					
(9,1)-(9,5)									X						X			X		X				X	

2.5 10 ORGANISMS, 5 AGAR SOLUTIONS

O-A Combina- tion	DAY																								
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
(1,1)-(1,5)	X									X								X							X
(2,1)-(2,5)	X	X									X							X							X
(3,1)-(3,5)	X	X									X							X							X
(4,1)-(4,5)	X	X									X							X							X
(5,1)-(5,5)	X	X									X							X							X
(6,1)-(6,5)	X	X									X							X							
(7,1)-(7,5)	X	X									X							X							
(8,1)-(8,5)	X	X									X							X							
(9,1)-(9,5)	X	X									X							X							
(10,1)-(10,5)	X	X									X							X							X

Elements of Variance - Covariance Matrix for Design 1.2

Elements of Variance - Covariance Matrix for Design 1.3

Elements of Variance - Covariance Matrix for Design 1.4

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
2	2	1	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
3	3	2	1	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
4	4	3	2	1	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
5	5	4	3	2	1	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21																																																																															